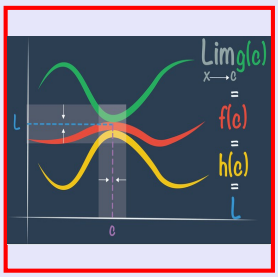
Math 261

Spring 2023

Lecture 6



Feb 19-8:47 AM

Evaluate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x)=ax^2+bx+C$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{a(x+h)^2+b(x+h)+\cancel{C}-ax^2-bx-\cancel{C}}{h}$$

$$= \lim_{h\to 0} \frac{a(x^2+2xh+h^2)+\cancel{bx}+bh-ax^2-\cancel{bx}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax^2}+2axh+ah^2+bh-\cancel{ax^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2ax+ah+b)}{\cancel{h}} = \lim_{h\to 0}(2ax+\cancel{ah}^{\to 0}+b)$$

$$= \boxed{2ax+b}$$

Suppose $2ax+b=0$

$2ax=-b$

$x=\frac{-b}{2a}$

x-coordinate of Vertex of $f(x)=ax^2+bx+C$

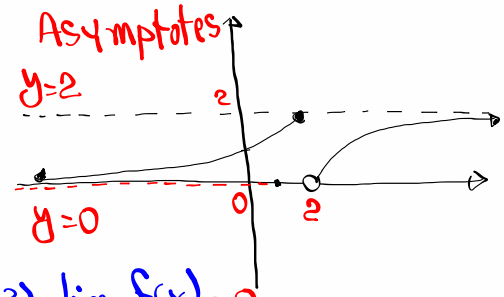
Feb 13-9:50 AM

Consider the graph below for the function $y=f(x)$

Horizontal Asymptotes

$y=2$

$y=0$



1) Domain $(-\infty,\infty)$

2) Range $(0,2]$

3) $\lim_{x\to 2^+} f(x)=0$

4) $\lim_{x\to 2^-} f(x)=2$

5) $\lim_{x\to 2} f(x)$ D.N.E.

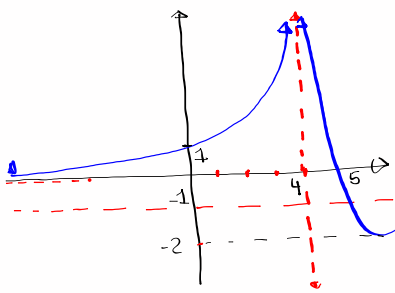
6) $f(2)=2$

7) $\lim_{x\to \infty} f(x)=2$

8) $\lim_{x\to -\infty} f(x)=0$

Feb 14-8:58 AM

Consider the graph of the function $g(x)$ below:



1) Domain $(-\infty,4)\cup(4,\infty)$

2) Range $[-2,\infty)$

3) All Asymptotes

V.A. $\Rightarrow x=4$

H.A. $\Rightarrow y=-1,\ y=0$

4) All intercepts

x-Int $(5,0)$ y-Int $(0,1)$

5) $\lim_{x\to 4^+} g(x)=\infty$

6) $\lim_{x\to 4^-} g(x)=\infty$

7) $\lim_{x\to 4} g(x)=\infty$

8) $g(4)$ undefined

9) $\lim_{x\to \infty} g(x)=-1$

10) $\lim_{x\to -\infty} g(x)=0$

Feb 14-9:06 AM

Given $f(x) = \dfrac{x^2-2x}{x-2}$

1) $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \dfrac{x^2-2x}{x-2} = \dfrac{3^2-2(3)}{3-2} = \dfrac{9-6}{1} = \dfrac{3}{1} = \boxed{3}$

2) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{x^2-2x}{x-2} = \dfrac{0^2-2(0)}{0-2} = \dfrac{0}{-2} = \boxed{0}$

3) $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{x^2-2x}{x-2} = \dfrac{2^2-2(2)}{2-2} = \dfrac{0}{0}$ I.F.

$= \lim\limits_{x \to 2} \dfrac{x\cancel{(x-2)}}{\cancel{x-2}} = \lim\limits_{x \to 2} x = \boxed{2}$

Feb 14-9:15 AM

Evaluate

1) $\lim\limits_{x \to 3} \dfrac{x-3}{x^2-5x+6} = \dfrac{3-3}{3^2-5(3)+6} = \dfrac{0}{0}$ I.F.

$= \lim\limits_{x \to 3} \dfrac{\cancel{x-3}}{(x-2)\cancel{(x-3)}} = \lim\limits_{x \to 3} \dfrac{1}{x-2} = \dfrac{1}{3-2} = \boxed{1}$

2) $\lim\limits_{y \to 4} \dfrac{4-y}{2-\sqrt{y}} = \dfrac{4-4}{2-\sqrt{4}} = \dfrac{0}{0}$ I.F.

$= \lim\limits_{y \to 4} \dfrac{(4-y)(2+\sqrt{y})}{(2-\sqrt{y})(2+\sqrt{y})} = \lim\limits_{y \to 4} \dfrac{\cancel{(4-y)}(2+\sqrt{y})}{\cancel{4-y}}$

$= \lim\limits_{y \to 4} (2+\sqrt{y}) = 2+\sqrt{4} = \boxed{4}$

Feb 14-9:24 AM

$$f(x) = \begin{cases} x-2 & \text{if } x<0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

$x-2$ | x^2

$\rightarrow 0 \leftarrow$

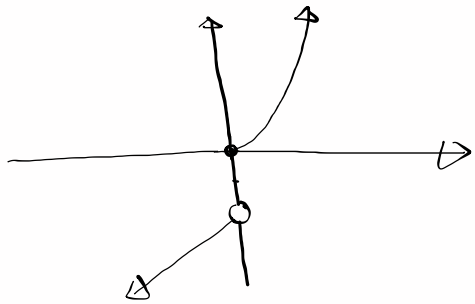
1) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x-2) = \boxed{-2}$

2) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = \boxed{0}$

3) $\lim_{x \to 0} f(x) = \boxed{\text{D.N.E.}}$

4) $f(0) = 0^2 = \boxed{0}$

5) Graph $f(x)$



Feb 14-9:32 AM

$$f(x) = \begin{cases} \dfrac{x^2-4}{x+2} & x \neq -2 \\ \boxed{K} & x = -2 \end{cases}$$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \dfrac{x^2-4}{x+2} = \dfrac{0}{0}$ I.F.

$= \lim_{x \to -2^-} \dfrac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = \lim_{x \to -2^-} (x-2) = \boxed{-4}$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \dfrac{x^2-4}{x+2} = \dfrac{0}{0}$ I.F.

$= \lim_{x \to -2^+} \dfrac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = \lim_{x \to -2^+} (x-2) = \boxed{-4}$

$\lim_{x \to -2} f(x) = \boxed{-4}$

Find K Such that

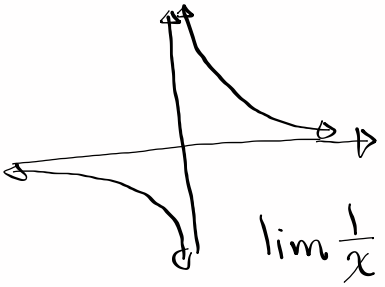
$f(-2) = \lim_{x \to -2} f(x)$

$\boxed{K = -4}$

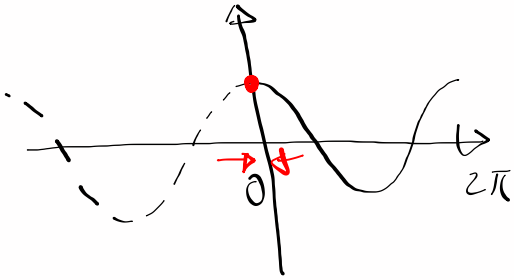
Feb 14-9:40 AM

Evaluate $\lim_{x \to \infty} \cos\left(\frac{1}{x}\right) = \lim_{t \to 0} \cos t = \boxed{1}$

$\cos 0 = \boxed{1}$



$\lim_{x \to \infty} \frac{1}{x} = 0$



Feb 14-9:45 AM

Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = \sqrt{x}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{x} + \cancel{h}^{1} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}} \checkmark$$

$$\frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \boxed{\frac{\sqrt{x}}{2x}} \checkmark$$

Feb 14-9:50 AM